JuggLing-a-Ding: Design and Development of a Musical Juggling Toolkit

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Figure 1: The table from a juggler's training session of *Concerto for Four Jugglers and A Piano.*

Abstract

The musical juggling artist Vincent de Lavenère has spent a decade exploring the possibilities and challenges of a novel musical instrument composed of acoustic musical juggling balls that each play a note when caught. We first report on results from an initial user study that identifies specific needs and requirements for musical juggling. We then introduce a notation system for musical juggling performances, together with *JuggLing-a-Ding*, a web component for prototyping interactive tools that support the different phases of artistic creation. We next describe a modular pipeline that begins with a musical juggling performance specification, computes event-based and physical models, and renders them in 3D. We then explain how our implementation leverages multiple web technologies for ease of deployment and integration into a rich ecosystem of components, devices and platforms, including VR. Finally, we discuss future plans for interactive tools and studies with artists that build on *JuggLing-a-Ding*.

Keywords

Musical Juggling, Music, Juggling, Notation, Score, Siteswap, Simulation, Web Component

1 Introduction

Juggling is essentially rhythmic and thus potentially musical. Many jugglers perform to live or recorded music. Some play a musical instrument while juggling[1], while others play music through the very act of juggling.

We are interested in juggling as an artistic practice that combines the visual spectacle of juggling with the creation of live music. Specifically, we have been collaborating with Vincent de Lavenère, founder of the *Chant de Balles* troupe, who has spent the past thirty years exploring acoustical musical juggling. Along the way, he invented new types of juggling balls. Some are filled with sand and sound like a maraca when caught or shaken. Others contain pebbles that roll between a pair of conjoint bells and play a note while spinning airborne. A decade ago, he introduced the even more complex and visually appealing *bell balls* (see Figure 2). These contain a tuned bell whose clapper is replaced by a weighted spring. Catching a bell ball rings the bell and plays a note. Together, a collection of bell balls comprise a novel musical instrument that a skilled juggler can use to perform a musical composition. Vincent de Lavenère has used this instrument not only as a solo performer, but also as a solo artist accompanied by a symphonic orchestra or as part of a concerto with four jugglers and a piano.

The instrument itself is deceptively simple, yet imposes peculiar constraints. First, the juggler must take into account a variable second-scale delay between the initial act of throwing the ball and the production of the note. Second, even simple melodies may require throwing balls out of order relative to the

[1] See for instance *Paul & Máx*

notes they produce, or putting down or picking up balls on the fly (see Figure 1). This requires pre-planning to coordinate throws and notes. Also, the juggler cannot practice by playing slowly, nor is it possible to read a score while playing. Together, these constraints make memorizing each performance extremely challenging, since the required anticipation prevents the juggler from relying on mentally hearing the melody. An additional technical challenge stems from the ear's sensitivity to rhythm fluctuations, which in turn requires unusually precise timing when catching each ball.

These inherent challenges call for designing tools that support artists as they compose, learn, train for, and perform with bell balls. As a first step, this article explores the problem of notating and animating musical juggling patterns, a core component for the development of future tools.

We first describe related work on musical juggling, as well as background on the *Siteswap* notation and software for traditional juggling. We then describe the results of an initial study that identifies specific needs and requirements for musical juggling. Next, we introduce an enhanced notation system specifically designed for musical juggling performances and present *JuggLing-a-Ding*, a web component for prototyping interactive tools that supports the different phases of artistic creation. We describe *JuggLing-a-Ding*'s modular pipeline and how its implementation leverages multiple web technologies to facilitate deployment within a rich ecosystem of components, devices and platforms, including Virtual Reality (VR). We conclude with a discussion of the limitations and directions for future research.

2 Related Work

We first describe research on the use of juggling to control music. We then explain multiple approaches for notating juggling patterns, with an emphasis on the *Siteswap* notation and its contribution to the emergence of juggling software.

2.1 Juggling and Musical performance

Various artists and researchers have explored juggling as a way to control electronic music [14, 16], including how to generate sounds by juggling [2], create rhythms from juggling patterns [4], or use spatial audio to sonify a juggling performance. Marshall [7] created a vision-based tracking system that records professional circus jugglers' movements to create live audio and visual elements within their performance. Özcan and Çamcı [18] describe juggling as a means to explore failure and playfulness in music performance, and how failure can enhance musical expression for beginning jugglers.

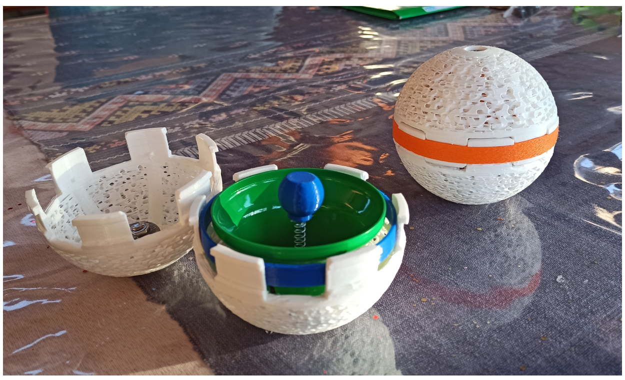

Figure 2: Two bell balls — the left ball is split open to reveal its tuned bell and weighted spring.

Each of the above systems use a similar process to produce sound: different types of sensors gather data from the juggler, including cameras, tracking systems, accelerometers embedded in the props or worn by the juggler such as electro-myographic sensors. These systems also record or infer juggling properties, such as the position, acceleration or orientation of the device or the juggler's hand or arm; the timing of each throw and catch; or the overall tempo or time in the air. Finally, the data is mapped to specific outputs, including lighting, pitch and sound effects.

Others, such as Vincent de Lavenère or the troupes *Les objets Volants* and *Kuutenkidou*, focus instead on acoustic devices. This focus stems from a quest for simplicity and transparency, to ensure that spectators can predict the outcome of each action, as well as the technical and artistic challenges imposed by this highly constrained design space.

2.2 *Siteswap* Notation

Siteswap [6] is both a family of mathematical models and notations for juggling patterns. They are similar to musical score notations, but encode throws instead of notes to play.

The simplest *Siteswap* model makes stringent assumptions about juggling: the juggler alternates between the left and right hand to a regular beat, catching and throwing at most one ball at a time. In this model, a juggling pattern is notated by a sequence of integers, one per beat. It encodes the *airtime* or *height* of the throw at that beat, indicating for how many beats the thrown ball stays airborne before being caught again. Typically the sequence is periodic and only one period is notated. Figure 3 shows the juggling diagram associated with the *Siteswap* sequence 423, where a ball is thrown on beat two from the right hand for 3 beats. This means that that ball will be caught with the left hand on beat five. The arcs drawn over each sequence illustrate what happens when a ball is thrown.

More complex models capture richer forms of juggling. For example, multiplex juggling [10] involves throwing and catching multiple balls from the same hand at the same time. Synchronous juggling — using both hands at the same time — can be combined with asynchronous juggling — alternating hands — and passing can express how multiple jugglers pass balls to each other.

Siteswap has been studied extensively as a combinatorial problem [9] to answer questions such as: How to test whether a sequence of integers is a juggling pattern? How to draw, count, or generate all juggling patterns? These questions have elegant and efficient solutions based on finite automata, constructed by taking "freeze frames" of a juggler (called *juggling states*) and

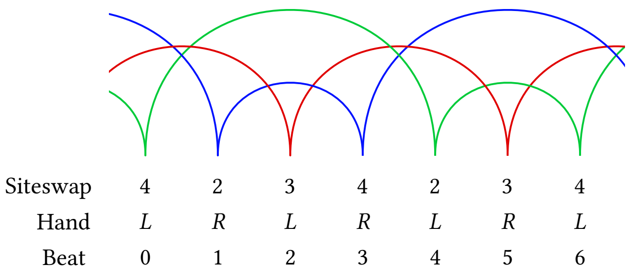

Figure 3: Juggling diagram of the *Siteswap* sequence 423. Colored arcs help track the trajectories of each ball.

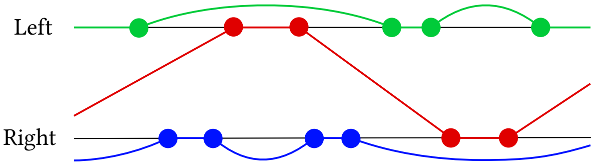

Figure 4: The ladder diagram representing the *Siteswap* sequence 423 shows how long each ball remains in each hand between throws and catches.

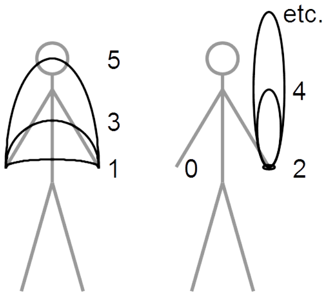

Figure 5: A throw trajectory based on its *Siteswap* value[2]

computing how different *Siteswap* tosses transform a juggling state into another.

2.3 Alternative Notations

Some notations complement *Siteswap* with movement annotations, such as tossing a ball from behind the back and catching it between the legs. For example, Mill's Mess State Transition Diagrams handle crossing and uncrossing hands, and can thus explain why the 3-ball cascade and the Mill's Mess are nothing alike despite having the same *Siteswap* sequence.

Other notations deviate further from *Siteswap*, straying from integer sequences altogether. Some jugglers find the causal diagrams developed by Martin Frost more intuitive than *Siteswap*, since they conceptualize throws as "making room for another ball that is about to fall". Graphic juggling notations and harmonic throws apply symbols to a musical staff, which further highlights the link between music and juggling. See Simu [13] for a more complete discussion of juggling notations.

Ladder diagrams, first introduced by Shannon in 1993 [12], offer an alternative description of juggling patterns (see Figure 4) that look down on a walking juggler and record whether each ball is airborne or held in the juggler's hand. Ladder diagrams are more expressive than *Siteswap* patterns because they capture the amount of time balls spend in each hand and permit throw or catch events to occur at any time. They also offer a simple visualization that supports reasoning about juggling patterns. However, they are harder to memorize and communicate verbally and become messy, e.g. when depicting multiple jugglers. If *Siteswap* is the equivalent of a musical score, ladder diagrams function as the counterpart of MIDI, which more precisely captures the timing of events. Balls thrown or caught are similar to notes being turned on or off.

[1]https://en.wikipedia.org/wiki/Siteswap — public domain.

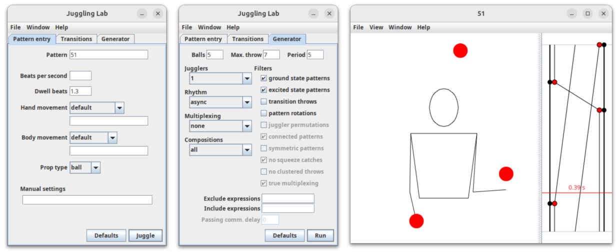

Figure 6: *JugglingLab*: Juggling pattern input (left), generator (center); and animator with ladder diagram (right).

2.4 Juggling software: Leveraging *Siteswap* for the casual juggler

The first juggling software programs were developed in conjunction with *Siteswap*. In 1988, Allen Knutson distributed *Juggle*, a juggling animator that takes a *Siteswap* sequence as input, on the *rec.juggling* newsgroup. In 1991, Bengt Magnusson, who produced the first article on *Siteswap* [6], programmed a generator that, given constraints such as the number of balls or the maximum height of a throw, yields all possible *Siteswap* patterns. Since then, many more animators and generators have been developed to visualize *Siteswap* and discover patterns, using more and more expressive *Siteswap* frameworks without changing the core notation[3]. The combination of these tools, together with the conceptual framework provided by *Siteswap*, has deeply influenced juggling practice by helping identify new patterns and fluid transitions from one pattern to another. The most well-known juggling program is *JugglingLab* (see Figure 6), an open-source project created in 1997. (See Section 5 for a description and how it relates to *JuggLing-a-Ding*.)

The success of these tools stems from both the simplicity and accessibility of the *Siteswap* notation, as well as the fact that the jugglers who created them were, as Candy puts it, "power users" [3] who created tools that appealed not only to themselves but also to jugglers in general. Of course, not all jugglers are familiar with this notation. Even so, novice jugglers usually learn *Siteswap* very quickly, since the airtime for each tossed ball intuitively matches a trajectory (see Figure 5). One could also argue that *Siteswap* is universal, since the notation was discovered independently by three different groups before it was published officially in 1989 [9].

Siteswap also appeals to teachers and jugglers who have used it as a method for engaging in mathematical thinking [8, 17]. In this vein, Vincent de Lavenère endorses the popularization of the mathematics of juggling.

2.5 Prior work on Vincent de Lavenère's musical practice

In 2000, before the bell balls were invented, Vincent de Lavenère premiered a musical show entitled "*Chant des Balles*", which also became the name of his troupe. He co-authored a book [1] that served as both an artistic and documentary project where he questioned how to capture musical juggling performances.

This led him to engage in a decade-long art-science collaboration with Florent Hivert and Nicolas M. Thiéry (the fourth author of this paper). Both are researchers in combinatorics at the LISN (formerly LRI) research lab at Université Paris-Saclay,

[3]See http://www.juggling.org/programs/ for a software list up to 1996.

and have over 30 years of experience as amateur jugglers. Together, they created the hybrid show *Musical Juggling, Automata and Combinatorics* in 2016 which uses juggling and *Siteswap* to teach the basic principles of modeling, finite automata and the scientific method to the general public.

Early explorations for this show involved drawing Ladder diagrams on the blackboard and tinkering with *JugglingLab*'s source code to model short musical juggling phrases. Kulinski and Moreau [5] continued this work by showing how to extend juggling models to a simple musical context, with efficient algorithms for composing patterns that play a desired melody.

3 Preliminary Studies

The work led for the aforementioned show demonstrated the potential of musical juggling notations and software for supporting a variety of artistic activities, facilitating composition and learning, sometimes becoming part of the performance itself. It revealed limitations of current juggling notation and software, especially with respect to expressiveness, flexibility, explainability and debugging feedback to tackle long, non-repetitive musical juggling performances. It also highlighted the need for prototyping tools that support different phases of the creation process, with a common underlying notation and animated visual representation. The next section describes a preliminary study designed to elicit requirements for such tools.

3.1 Methodology

In addition to weekly or monthly meetings with Vincent de Lavenère, we followed a participatory design process with novice and professional jugglers to explore improved ways of notating and animating five existing musical juggling performances, which included *Concerto for four jugglers and a piano*, a two-juggler rendition of *The Blue Danube* (see Figure 8-left), and the solo piece *Petite Fleur* (see Figure 8-right). Section 3.2 describes how the latter influenced our understanding of tempo.

We also observed and informally documented training sessions and performances organized by Vincent de Lavenère:

- 1 Week-long training residency for *"Concerto for four jugglers and a piano"* (December 2024);
- 4 Workshops to teach orchestra students how to juggle (March 2025);
- 2 Science fairs and workshops for high-school students (Late 2024).

Over time, our focus shifted from the basic question of how to model musical juggling to the problem of how to assist artists. We conducted semi-structured interviews with three of the four professional artists involved in the *concerto*, one year after their residency. Each had over 30 years of juggling practice and was equally familiar with musical props besides having different musical experience. We were interested in how they had trained for the original *concerto* performance, as well as how this experience affected their subsequent musical juggling practice, including practice exercises and any resources they created and used. A full analysis of these interviews is beyond the scope of the present article. However, we present several insights that have proven valuable for the dual challenges of creating a new musical juggling performance and honing the skills required to perform it successfully.

3.2 Example: Understanding juggling tempo

Modeling the *Petite Fleur* solo led to our first extension of *Siteswap*. The piece is challenging because it involves regular rhythm changes. We first observed how Vincent de Lavenère performed the piece with its off-beat throws and catches, especially how he transitioned between fast and slow rhythms. We then explored how to capture the complexity of the piece with a notation.

Unlike music which allows for tempo to change, *Siteswap* assumes a constant beat and rhythm. We first modeled coexisting individual patterns with their own constant rhythms, which is possible per say. But balls thrown at some tempo and caught with another can exhibit non-integer airtime.

We then tried Roudaut's [11] approach, which involved extending *Siteswap* such that the overall constant beat is divided by a particular factor. This ensures that all off-beats fall on a beat: the juggler can increase the height of each throw or add empty throws to compensate. For example, the *Siteswap* pattern 3 at one tempo can be written as $(6x, 0)(0, 6x)$ at twice that tempo.

However, when we observed how Vincent de Lavenère taught this piece to other jugglers and in subsequent interviews, it became clear that the above approaches do not align well with how jugglers think about their performance. They consider chaining half or quarter notes to be juggling the same pattern at a different tempo, and handle the transitions very intuitively. After several iterations, we decided to add a "tempo track" to to clarify the transitions while keeping intuitive *Siteswap* patterns (See the revised notation of *Petite Fleur* in Figure 7.)

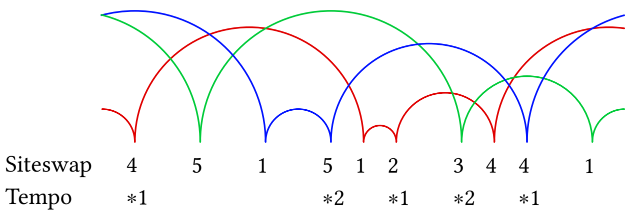

Figure 7: *Siteswap* is augmented with a tempo track to notate a sequence from *Petite Fleur.*

4 Results

4.1 Musical Juggling characteristics

The process of modeling musical juggling performances led us to identify the following set of key characteristics specific to performing with bell balls that distinguish it from traditional juggling and *Siteswap* notations:

- *Distinct balls*: Each ball has one or more distinguishing features, e.g. tone or color.
- *Toss and Catch order*: Since the balls are distinct, the order in which they are tossed and caught matters. In addition, when multiple balls are held in the same hand, it is easier to throw or catch from specific locations. This may require conscious movements of the balls in hand.
- *Limited grip*: The artist can only hold a limited number of balls in each hand. Similarly, the complexity of the involved juggling patterns imposes a strict limit on the number of balls (typically 2-5) that can be juggled simultaneously.
- *Large set of balls*: Playing a variety of notes requires juggling just as many balls. Given their limited grip, the artist must pick up and lay down balls on the fly, which in turn

requires pre-planning and memorization of a *predictable* ball layout. (Figure 1 shows a table with a set of currently unused balls).
- *Distraction strategies*: If grabbing new balls is impossible or takes too long, the artist may sing or let another juggler play to continue the music.
- *Silent catches*: Bell balls were designed to enable silent catches by dampening their impact. Artists take advantage of this to introduce bell balls one sound at a time at the beginning of a show, to skip a note for a particular melody and to continue juggling during long silences.
- *Pattern variety*: Depending on the music being played, patterns may be decomposed into repeated shorter sub-patterns that are especially useful for unique tricks. Conversely, when playing melodies, patterns tend to be longer with far fewer repetitions than typical juggling. Either way, patterns need to be understood at the scale of the whole music or performance.
- *Tempo*: The number of throws per minute may vary over time or across jugglers. (See subsection 3.2).

4.2 Design Recommendations

Based on our preliminary study and the above characteristics, we suggest the following recommendations for designing a musical juggling notation system and animator to support the creation of future tools.

- *Easy to grasp, possible to master*: The target users have diverse profiles, including amateur and professional jugglers, musicians, researchers, and the general public. The tools should include a low barrier to entry, with an intuitive mapping to basic juggling practice, while permitting the creation and training of more complex patterns.
- *For humans, by humans*: Similar to a music score, the notation is meant to be written and read by humans, and only incidentally by computers. This requires a concise, non-redundant, and flexible approach with multiple levels of detail, from a simple pattern indication to a full, formal recording of a performance, with complete control over the interpretation. For example: "On beat 3.1 Alice throws the blue D ball number 3 of radius 4cm from position XXX with airtime 2.7").
- *Accessible*: Tools must be available on any device in a variety of settings for a diverse set of target users. The tools and resulting artifacts must also be easy to share.
- *Composable and Sustainable*: The notation and animator are meant as a robust foundation for building research prototypes that explore how to support the different phases of artistic creation. However, they can also serve as standalone products that can be used over time and shared within the community, ideally for future generations of musical jugglers.
- *Extensible*: Programmers should be able to easily extend the notation and animator to facilitate note taking and as a memory aid, to add new levels of abstraction, and to accommodate future art-science collaborations, for example by handling polyphonic accompaniment or music tailor-made for juggling.

5 *JuggLing-a-Ding*

We introduce *JuggLing-a-Ding*, a web component for prototyping musical juggling performances, and *Musical Siteswap*, an enhancement to the *Siteswap* notation. We outline *JuggLing-a-Ding*'s main features; explain how to describe a performance; and show how this description can be turned into a model that can be animated in 3D. We conclude with a short description of the implementation and directions for future work.

5.1 Main features

JuggLing-a-Ding (see Figure 8) is a web-based musical juggling component. Figure 10 shows the overall architecture, which takes as input a JSON specification of a performance or any lower level description and animates a 3D rendering of it, with sound. This specification can be exhaustive or partial thanks to default values or automatic inference for most fields. *JuggLing-a-Ding* is designed to facilitate writing and debugging performance descriptions. It gracefully handles errors related to incorrect descriptions by logging them and visually highlighting them in the animator. The animator can be synchronized with external sources, such as a clock, video or music, and emits juggling events that can be bound to callback functions.

The code is open-source and available on a public forge[4] and will also be published as a pre-packaged library. A standalone version with a minimal UI for playback and example performances is available on a static website[5] and can be downloaded as a web application for offline usage. *JuggLing-a-Ding* can be embedded in any website, including as part of an online presentation, by using an `iframe`. In the latter case, the performance description can be specified in the query string of the URL.

5.2 Performance description

The three components of a performance description are the performance geometry, the object visual descriptions and the musical juggling score.

The *Performance Geometry* describes the positions and dimensions of key elements in the performance:

- Sizes and positions of the jugglers and tables;
- Radii of the balls;
- Table layouts, to indicate where balls can be placed or taken from; and
- Hand layouts, to control where balls are positioned when multiple are held.

The *Object Visual Descriptions* describes how the jugglers, balls and tables should look like, to create 3D meshes from.

The *Juggling score* describes the musical juggling pattern at the scale of a whole performance, including:

- *List of musical balls*: includes bell balls, percussive balls, and balls that play a note while airborne;
- *Starting configuration*: identifies locations of balls on the table(s) and in the jugglers' hands; and
- *Phrase Sequence(s)*: describes musical juggling phrase(s) for each juggler.

Structuring the score into musical juggling phrases facilitates writing longer patterns. Phrases are defined by their start time, a base tempo, a set of balls that should change position before the beginning of the phrase, and a *Musical Siteswap* pattern (see Figure. 9).

Start time and base tempo can be expressed in multiple units: either in a juggler's internal (i.e. local) beat system, or more broadly in seconds, or in a global beat system common to all

[4]https://github.com/kunchtler/mj-lib
[5]Hosted at https://leokulinski.fr/juggling-a-ding with code available at https://github.com/kunchtler/juggling-a-ding-website

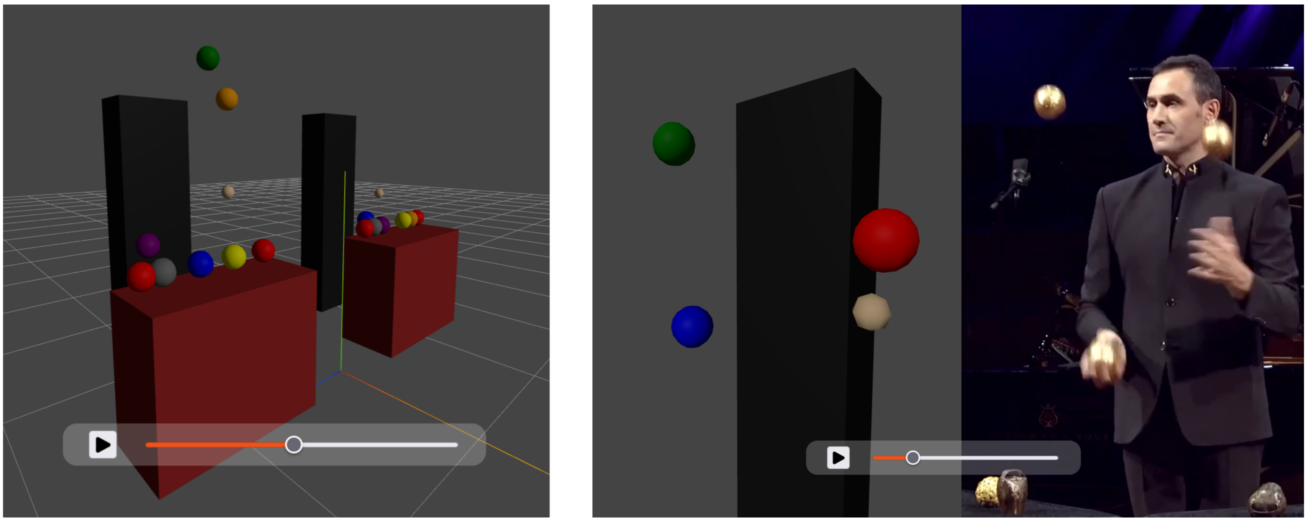

Figure 8: *JuggLing-a-Ding*, used to animate a performance with tables and to synchronize with a video.

```
{
    "time": {
        "type": "localBeat",
        "value": "2"
    },
    "setup" : {
        "hands": [["Do", "Re"], ["Si"]]
    },
    "tempo" : {
        "type": "perSecond",
        "value": "180"
    }
    "pattern" : "3 2s 3 [Do1 Si3] Tempo*2 3^10
        (441)^6"
}
```

Figure 9: A juggling phrase specified in *JuggLing-a-Ding*'s JSON notation.

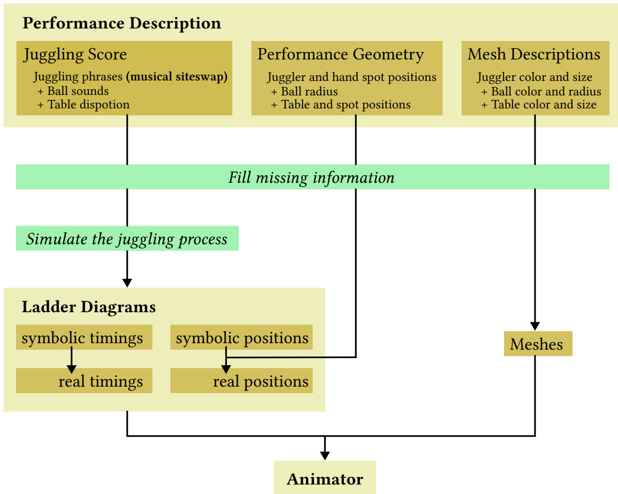

Figure 10: The architecture of *JuggLing-a-Ding*

jugglers, which facilitates passing balls. To help coordinating with existing musical scores, global beats can be further regrouped into measures. They are defined by their time signatures which describe the number of beats per measure.

Musical Siteswap is based on the *Siteswap* notation and is designed to capture how jugglers throw and catch musical balls. It takes advantage of jugglers' familiarity with *Siteswap*, and supports most common extensions: multiplex, combined synchronous and asynchronous notation, crossed throws, repetitions, passing.

Musical Siteswap adds the following capabilities to *Siteswap*:

- Holding balls in the hand;
- Putting down and collecting balls from a table;
- Manipulating held balls;
- Specifying which ball to toss when a hand holds multiple balls;
- Catching balls silently; and
- Changing the tempo.

We included two ways of manipulating the tempo track because juggling includes two distinct interpretations of tempo that cannot be distinguished with a musical phrase alone. Jugglers must be able to affect the overall pace of the piece, but also play offbeat notes to be able to play the melody as intended.

5.3 Pipeline

We briefly describe how a performance description is transformed into a ladder diagram that can in turn compute the positions of each moving part (see overall structure in Figure 10). During this process, we differentiate symbolic time, e.g. "On beat 3", from real time, e.g. "At 1.2 seconds", and symbolic position, e.g. "Where the juggler's left hand would catch a ball", from real position, e.g. "At coordinates (1, 0.2, 1)".

Starting with the *juggling score*, we fill out each optional piece of information from the description with default values and parse the *Siteswap* strings. We then convert each different timing systems into a common unit: global beats.

At this point, we know when tosses occur and for how long balls fly, but not which balls are thrown, where from and where to. Similarly to the juggling states in juggling automata, we keep track of where each ball is — airborne, held or on the table — and simulate tosses, catches, and interactions with the table, global beat by global beat. This process allows discovering whether a pattern is ambiguous or impossible, for example when throwing a ball that does not exist. It results in a ladder diagram with symbolic positions and symbolic times, and

The next step is to use the performance geometry to create a ladder diagram with real positions and real times. We compute reasonable timings so that balls remains in hand long enough between two events.

Finally, we can compute the position of each moving object at any time simply by examining its previous and subsequent events. For example, we have physical equations to compute the ball's trajectory given the real times and positions of its toss and catch. A hand's position can be calculated from a cubic Bezier curve whose control points are given by the positions and velocities of balls caught and tossed.

5.4 Implementation

The current version of *JuggLing-a-Ding* consists of 13k lines of TypeScript code, half of which handles conversions between higher and lower-level descriptions. We developed it as a web application for portability and tested it on *Firefox* and *Google Chrome*, both on mobile and desktop platforms. As mentioned earlier, the animator can be embedded in any website using an `iframe`. *JuggLing-a-Ding* is also a Progressive Web Application (PWA), i.e. it can be turned into a standalone application, for example on a mobile device.

The lexer and parser for the *Musical Siteswap* notation are specified using ANTLR, which can generate code in different programming languages from a grammar specification. The user interface uses React. The 3D rendering by the animator uses the ThreeJs library. The animator also runs in VR using the React-three-fiber library, which combines ThreeJs with the React syntax and render engine.

5.5 Discussion and Future Work

This article describes the rationale and design of *JuggLing-a-Ding*. Vincent de Lavenère has successfully showcased it in several recent shows and amateur juggling workshops. The notation itself has mostly been edited by its creators, but is readable by jugglers.

We show how *Musical Siteswap* successfully describes complex, existing musical juggling performances that feature all of the characteristics outlined in Section 4.1, and incorporates the following recommendations from section 4.2:

- *Musical Juggling characteristics*: All the outlined characteristics are present in performances we've been able to model.
- *Easy to grasp, possible to master & For humans, by humans*: Early feedback from users suggest that the animation provides an easy-to-navigate, realistic and intuitive account of the performances. The system lets users describe musical performances with as little or as much detail as needed. This makes it easy to model simple performances, but still allows modeling complex ones.
- *Accessible*: The implemented system is available on multiple devices, both online and offline, for multiple target audiences in multiple settings.
- *Composable and Sustainable*: We followed traditional code and Open Science best practices. A student intern was able to successfully integrate the animator into a Virtual Reality environment, indicating its flexibility.
- *Extensible*: After several iterations, *JuggLing-a-Ding*'s architecture converged to easily adapt to new features in the notation.

Even so, *JuggLing-a-Ding* has several limitations that suggest directions for future work. First, the notation is still subject to change depending on future artistic exploration. Jugglers may want more flexibility in specifying how they perform catches and tosses, which depends on their own preferences and the size and amount of balls they are holding. Second, while our model was designed to be as constraint-free as possible, musical juggling scores imposes additional constraints focusing on Vincent de Lavenère's performances and not on everything possible with bell balls. For instance, polyrhythmic juggling, where each hand catches and throws at a regular tempo different from the other one, is cumbersome to express in our notation. Similarly, our model isn't concerned with jugglers moving during the performance, which may be more prominent with other juggling troupes.

Finally, we need to conduct studies to assess the power, simplicity and flexibility of juggling musical scores; and of *JuggLing-a-Ding*'s ability to support the full range of musical juggling activities, including:

- *Composition*: *JuggLing-a-Ding* could be extended into a full creativity support tool that displays a musical score next to a musical juggling score and its animation. Juggling patterns could be generated by manipulating music and vice versa, using Python or Javascript-based computational music and juggling libraries.
- *Gradual training*: *JuggLing-a-Ding* could be incorporated into an Extended Reality (XR) environment to help jugglers experiment with and memorize new patterns, for example to collaborate with virtual jugglers or to practice in slow motion.
- *Resiliency training*: *JuggLing-a-Ding* could compute and visualize ways to start a pattern in the middle, without having to perform it from the beginning. This would support learning long phrases or training how to better recover from a ball dropped during a performance.
- *Recording*: *JuggLing-a-Ding* should include the ability to annotate scores and support custom note-taking, inspired by the ones jugglers already produce on paper.

These may lead to different designs that address the specific needs of each activity. For example, the animator is useful for composition and training, but is less relevant for live performance, which relies primarily on the juggler's memory. A future tangible tool could offer haptic feedforward or feedback to help the juggler select, throw and place balls in real time.

We are also interested in how domain-specific notations can be applied to other constrained instruments beyond musical juggling, and how it affects their creative use. Future work will explore how mathematical modeling can shape both the design and use of such instruments, and allow us to better understand, master and teach them. A similar example can be found in *change ringing*, a practice from 17th century England that involves ringing heavy bells with significant rotational delay. A systematic notation was developed which shaped how it could be played, slowly changing the order bells are rang to iterate through all possible mathematical permutations [15].

6 Conclusion

We are interested in the problem of how to help artists compose and capture musical juggling performances using a novel musical instrument created by Vincent de Lavenère: *musical bell balls*. Based on a preliminary set of interviews and observations with professional and amateur jugglers, we identify a set of their key features including distinguishable balls, in-hand ball manipulation, ball swapping, and tempo changes; as well as specific requirements for a notation system and animator.

We introduce *Musical Siteswap*, an extension of the *Siteswap* notation system we designed for musical juggling, which includes a musical juggling score, a performance geometry, and object visual descriptions. We describe the design and implementation of *JuggLing-a-Ding*, a web component that takes a musical juggling performance description as input and turns it into an interactive 3D animation. *JuggLing-a-Ding*'s architecture is based on a flexible, modular pipeline that first transforms the initial description into an event-based and a physical model, and then renders it. This pipeline allows users to input and customize data at any stage of the process. *JuggLing-a-Ding* offers a foundation for composing and exploring novel musical juggling performances, as well as helping artists capture and disseminate their compositions and performances. We also argue that this approach suggests directions for developing novel compositions that integrate music with physical, time-based activities.

7 Ethical Standards

All of the code developed in this research follows NIME's design guidelines, including for accessibility and inclusion. We include the URL for all references to web resources and include the date of last consultation in the final version. We collected informed consent from all participants, and from persons appearing in the photographs. Generative AI was used to help with the grammar and editing, but then thoroughly reviewed by a native English speaker.

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