Knotty Oscillator: Breaking knot topology for a new physically-inspired sound generator

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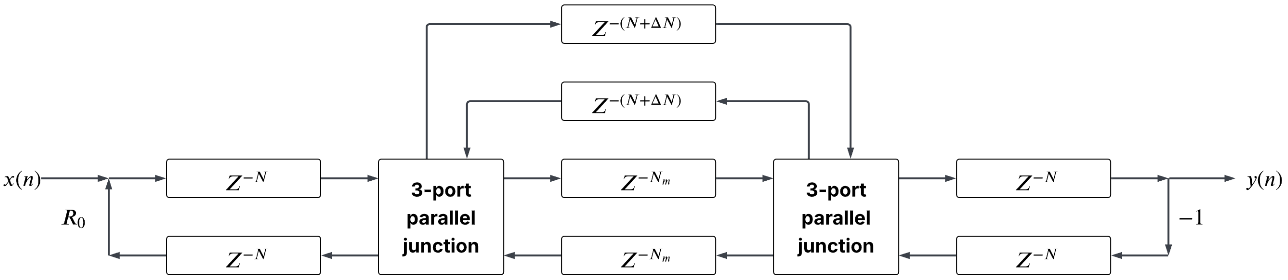

Figure 1: A knot-inspired waveguide with 3-port junctions - is the model we are exploring in this work.

Abstract

In this paper, we present a new type of 1-D waveguide oscillator inspired by knots, mathematical knot theory, and physical modeling. The main idea behind this study was to model a self-intersecting resonator, imagining it as one crossing of a knot. The number of crossings is known to be an invariant of knot class, which means that we could potentially use knot theory to distinguish variations of such oscillators, while using all the flexibility of geometric parameters. The problem was that the topology of mathematical knots forbids intersections at the points of crossing, and without intersection any knotted waveguide is just a tube. That is why we had to break knot topology to model one crossing as a chain of two three-port junctions. In this text we describe the concepts, the speculations and show practical results, including resulting VST-plugin.

Keywords

computer music, knots, NIME, topology, geometry, waveguides, physical modeling, real-time, VST

1 Introduction

If we try to imagine the least complex abstraction in three-dimensional space, we would probably arrive at a dot. A dot describes a position. A next level of complexity is a line connecting multiple dots. A line can describe a trajectory or a path. When such a line in three-dimensional space becomes a physical object (for example a thread or filament) and enters the real world of mass, volume, and forces, an important feature often emerges: the curve becomes self-avoiding. In topology this is formalized as an avoidance of self-embedding, meaning the curve can intersect it's own path, but can not merge into itself.

Within the enormous set of all possible tangled configurations of such elongated objects in 3D space, there exists a subset of knotted configurations. Each knot type is distinct, and in some cases - such as in protein molecules or marine knots or types of stitches - these differences can determine differentiated functional properties. By knotting a thread - we literally program it to perform different functions. At the same time, looking at it from another position, we can say, that we record the kinematics of our hand movements into the material of a rope or a filament. A spatial trajectory can thus encode information and help to execute a deterministic functions, and in the case of knotted paths this information is organized into a well-developed classification system with its own invariants and mathematical tools. In this sense, knots constitute a distinct and structurally meaningful phenomenon, which is fundamental in nature [11, 37].

In the physical world, one of the main features of a knot is its ability to hold itself. By adding a simple yet stable and distinct level of complexity to a basic morphology, - knots are often perceived as aesthetically pleasing [37]. For this reason, they have been used not only as configurable functional machines, but also as some of the most accessible decorative forms since the dawn of humankind.

Becoming more recognized in last hundred years [35], knots are subjects of interdisciplinary studies. They find applications in physics, mechanics, cultural studies, biology, etc. [2, 16]. Collaborations across zoology, computer science, material research, and robotics explore the unique capabilities of knotted living organisms [8, 26].

1.1 Knots in NIME

Computer Music studies and particularly NIME community has a long history of interest in entanglements, weaving, e-textiles, strands and various kinds of flexible morphologies [15, 17, 19, 24, 38, 39]. This interest has recently been reinforced with artistic, conceptual and technological research.

For example, The RUDIMENTS project investigates the "entangled" nature of musical instrument design, interweaving physical and conceptual braids. This group is not particularly researching

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properties of knots, but looks into entanglements under various perspectives [4].

Another collective, named the Khipunks, is a branch of wider movement of Neokhipukamayoqs - a revival of traditional culture of Khipu - Andean knot-making. They "decipher and code the content of data with knots, within the rope-based devices known as Khipus, memory-storing instruments" [9, 10, 19]. This group does focus on knots mostly as semantic devices embedded in particular cultural tradition and context.

Kasich previously published an overview of possible applications of knots in computer music, including NIME [20]. Our paper provides a deeper investigation into one of the specific instances outlined in that work. In particular, we draw inspiration from the physical and mathematical properties of abstract knots within a specialized area of NIME: digital synthesis and digital signal processing (DSP).

2 DSP inspirations from knots

Knots as physical entities are fascinating, and non less inspirational are their mathematical abstractions. At the same time the distinction is important.

2.1 Knots as mathematical objects

It is impossible to outline even a basic introduction to mathematical knot theory in frames of this paper. Therefore we can direct to sources: [2, 23, 29, 35]. Let us mention just a few important features, which distinct mathematical knots and are important for our explorations.

- `Mathematical knot` - is a closed polygonal curve in R^3 [23]. The most basic is the "trivial knot" or "unknot," - is simply a circle in 3D space.
- `Topology vs Geometry` - are two perspectives on mathematical knots, which can be combined or isolated. In Topology we look for equivalence under continuous deformations (identify types and classes), while in Geometry we consider measured features (like size or distance). Numerical apparatus is more applicable through geometry.
- `Invariants and Crossings`. Knot invariants are properties that remain unchanged under continuous deformations. They are topological tools used to identify and classify knots and form the basis of knot-specific calculations. There exist many different knot invariants. One of the most intuitive is the crossing number, defined as the minimal number of crossings over all possible planar diagrams of a knot. For example, the unknot has crossing number zero, while the simplest nontrivial knot - the trefoil - has crossing number three. There are no nontrivial knots with one or two crossings, since any such diagram is topologically equivalent to the unknot.
- `Typologies`. As said, there are many types of knots, based on plethora of invariants. One important group is Prime Knots - they are the basis of all other knots. 352,152,252 distinct non-trivial prime knots have been tabulated by year 2020 [5]. And another group important to us - Torus Knots - the symmetric topologies, which could be embedded on a surface of a torus. For example, a Trefoil - is the simplest prime torus knot.

2.2 Knots as oscillators

Intersection of topology, geometry and DSP - is a well known research field, involving interconnections of harmonic patterns in sound and visuals, mapping of parametric synthesis on topological structures, modeling analogue and digital hardware, dynamic systems and more [12, 13, 27, 28]. For example, Essl [14] showed the possibility of synthesis by deforming oscillatory signals, topologically mapped on circles.

Because all mathematical knots exist on closed curves - they can be computed using Fourier series [3, 5, 36]. But some are easier then others. For example, due to main attribute of Torus knots - embedding on torus - we can use a finite series with just two parameters [36]. Written in a more convenient way - the set of equations for three coordinates looks like this:

$$\begin{aligned} x &= r\cos(qt) \\ y &= r\sin(qt) \\ z &= -\sin(pt) \end{aligned} \tag{1}$$

where

$$[r = \cos(pt) + 2] \text{ and } [0 < t < 2\pi]$$

In this system: q - determines a position around the central hole of the torus (Longitude); p - describes the position on the surface through the hole of the torus (Meridian).

All of the above means, that any knot can be represented as three banks of oscillators with additive synthesis through Fourier series. And Torus Knots would be the most compact way, because it is possible to introduce coupling via amplitude modulation, where r (inverted and shifted version of z) - would be a shared modulator between x and y.

The working version of such a Torus Knot Synthesizer (Fig.2, 3) has been realized, using web tools (html, css, three.js, web audio API) and is available online: https://kasich.org/apps/torus-knot-synth/.

Parameterization of Torus Knots in audio format as 3 coupled oscillators:

Oscillator 1:
x = r * cos(p*t)
Oscillator 2:
y = r * sin(p*t)
Oscillator 3:
z = -sin(q*t)

where [r = cos(q*t) + 2] is the modulator
and range is [0 < t < 2π]

P (Meridian): 2.00
Q (Longitude): 3.00
Base Freq (Hz): 120
Tube Thickness (visuals only): 0.07

TREFOIL
T(2, 3)

Figure 2: An online Torus Knot Synth, synthesizing a Trefoil knot with three coupled oscillators. On the left corresponding spectrogram and waveform can be observed. A number of controllers on the right allow to gradually change p and q parameters, while experiencing change in sound and visuals.

Important to state, that in this version of the synthesizer all three oscillators sound mixed together in mono, without any amplitude coefficients. We consider this to be the most trivial case of knot-inspired synthesis. But what can be less direct and more inspirational?

2.3 Breaking knot topology

Indeed, if to ask a DSP professional about their first associations coming to mind, while discussing elongated topologies - additive synthesis would not be the first. Perhaps the more obvious one -

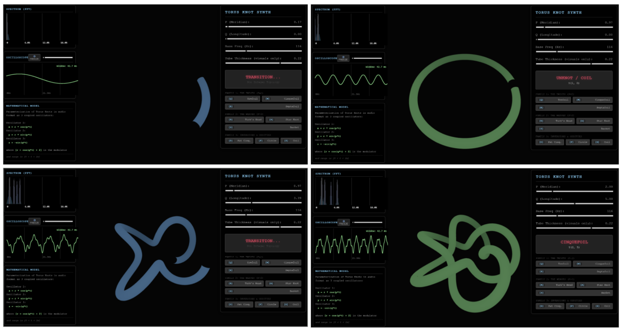

Figure 3: Demonstration of formation of Cinquefoil knot. Upper left: slow raise of q. Upper right: q approaches 1. Lower left: slow raise of p, while q is low. Lower right: q reached 2, p reached 5 - Cinquefoil is formed.

is a chain of digital filters in physical modeling. Those are used to model sound emergence in abstracted elongated objects like strings and tubes, adherent in their characteristics to the objects in physical world. In fact the term "topology" is widely used to describe various types of digital waveguide arrangements - the wide-spread technique in physical modeling [31, 33].

Taking this "visual" association - we can try to describe a "knotted digital waveguide".

The simplest waveguide (Fig.4) is a bidirectional delay line, that "can model any one-dimensional linear acoustic system such as a violin string, clarinet bore, flute pipe, trumpet-valve pipe, or the like" [32].

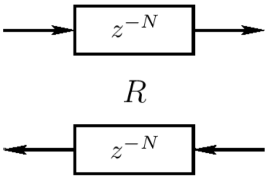

Figure 4: A simple digital waveguide with N samples long delay lines and wave impedance R (taken from [32]). This would be abstractly equivalent to a topology of one infinite string, or a tube.

If we approach this idea from the knot theory perspective - we need to focus on a particular knot invariant - for example the number of crossings. It may become a parameter for a future synthesis engine. This narrows the task to an attempt to model one distinct crossing. The obvious problem here is that the crossing, while being a significant entity for knot topology, - has no influence on topology of waveguides. This is because, as stated earlier, knots are defined through avoidance of self-embedding - the intersection of a curve is only possible without merging into itself. And if the curve (or a pipe or a tube) is not merging into itself - it is isomorphic to a straight waveguide (Fig.4).

That is why, to actually introduce the crossings into our physical modeling system - we have to break the knot topology and allow crossings to merge. We start with a simple finite tube of a certain length. We bend it to reach the crossing (required to identify any knot). And then we go one step further and merge the tube into itself at the point of crossing. Starting this stage - we no longer strictly follow knot topology, but we work with an unusual type of digital waveguide, inspired by knots. It looks more like a loop, and topologically isomorphic to a torus, while in the world of physical modeling it is important, that we do not form it into a torus, and keep the ends of the tube free. That is because one end allows us to trigger the system with input and introduce a *reflection coefficient* R_0, and the other - to register the output and invert the signal (see Figure 5).

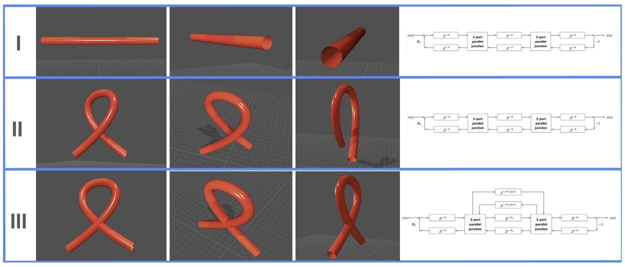

Figure 5: Three phases of the loop formation (please zoom in for better view). I - is a simple tube. It can be described as a basic linear waveguide (the right-most column represents the diagram). We add two 2-port parallel junctions to keep it consistent with further formation. II - is the crossing, which would be significant for knot topology, but is still a basic waveguide in the physical modeling domaine. III - the merged crossing, which becomes a looped waveguide with a chain of two 3-port parallel junctions.

The point of merged crossing can be described as a chain of two three-port parallel junctions. The full diagram of such system is in Figure 1. In the following sections we describe the modeling and tests of such system as a sound oscillator.

3 Knotty Oscillator

In order to explore the sonic palette of the structure, we measured and listened the impulse response of a one-crossing structure with various internal (length of the middle waveguide section) delay values N_m. In addition, the spectra of each simulation is compared to the more typical straight-line waveguide used to simulate strings and tubes [30], to measure the sonic impact of working with a more complex structure.

A diagram of the tested structure is shown in Figure 1. It includes a reflection coefficient R_0 at the input, with a value of $R_0 = 1.0$ for our tests, and a closed loop inverting the signal phase at the end. Since we want to study the pure impulse response of the structure, filters to model wall loss [1]; open-end and bell reflections [6, 22, 33]; and non-linear responses [18, 25] where not included. In addition, to preserve the overall delay length of the knot structure we define $\Delta N = |N - N_m|$ as the difference between the length of the middle section and the remaining delay lines, and use that value to compensate for the overall size of the structure.

The delay lines are interconnected by means of two 3-port parallel junctions. The value p_i^- going out of the port $i \in \{1, 2, 3\}$ is calculated using the following formula:

$$p_i^- = p_J - p_i^+$$

were p_J is the total pressure at the junction, borrowing language from acoustic tube modeling:

$$p_J = \frac{2 \sum_{i=1}^{3} p_i^+}{3}$$

Parallel junctions with 3-ports have been previously used to model tone holes in acoustic wind instruments [21]; oral and nasal airways [7]; and the avian syrinx [34].

For the comparisons we analyze the impulse response both in time and frequency domain, as well as perceived auditory differences by exporting the corresponding audio file. Finally, an estimation of the fundamental frequency f_0 is performed over the spectra by measuring the distance between the peaks. That way we can measure if the changes in structure also introduces changes in perceived pitch. All the experiments were implemented in Python with a sample rate $f_s = 96$ kHz and a duration per generated signal of $t = 1.0$ seconds.

3.1 Straight-line vs Knotty

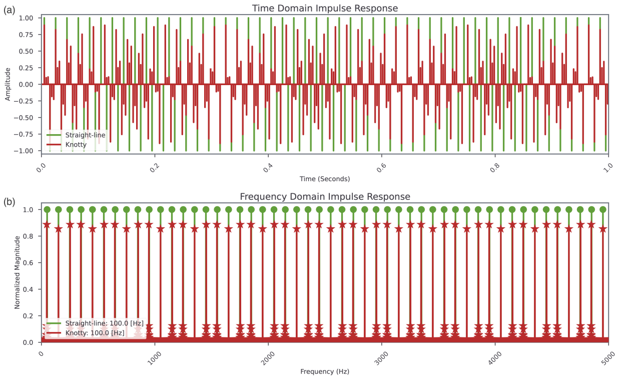

Figure 6: Comparison between straight-line and merged-cross waveguide structures for a delay length of $N = N_m = 160$ samples and a duration of $t = 0.5$ s. (a) Impulse response in the time domain. (b) Impulse response in the frequency domain up to 5000 Hz and estimated fundamental frequency f_0.

First, we compared the impulse response between a straight-line waveguide (See Figure 5 I, II) and a merged-cross one (See Figure 5 III), both with a delay of $N = 160$ samples and a mid section of the same length $N_m = 160$ samples for the crossing structure. We see that the position of the harmonics is almost preserved, and the estimation of f_0 coincides. Despite of that, the amplitude of the harmonics differs, and we can observe in the time domain plot that the reflected impulses of the merged-cross system do not align with the straight-line waveguide impulses. In addition, the signal of the crossed waveguide shows a periodic amplitude modulation in the time domain.

3.2 Comparison between several values of N_m

Figure 7 shows the frequency domain impulse responses of merged-cross structures with $N = 160$ and 3 different values of N_m, compared to the case $N_m = 160$, i.e., compared to when all delay lines have the same length. We observe that even though the harmonic peaks does not move far from their original position, different values of N_m produce frequency envelopes (or formants) in the magnitude of the peaks, changing the timbre of the output sound. In addition, the cancellation of certain peaks introduces deviations in the perceived pitch of the sound, as shown by the fundamental frequency estimations for each structure.

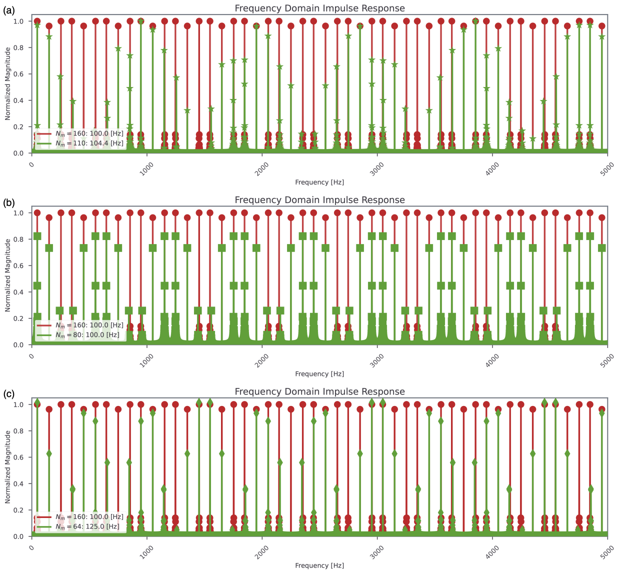

Figure 7: Comparison plots for the frequency domain impulse response of merged-cross structures with $N = 160$ and different values of the middle section length N_m, up to 5000 Hz. We use $N_m = 160$ (i.e., all delays of equal length) as a spectra of reference. (a) $N_m = 110$. (b) $N_m = 80$. (c) $N_m = 64$.

In order to study in detail the perceived pitch of the waveguides, Figure 8 (a) shows the fundamental frequency estimation for a merged-cross structure with $N = 160$ and several integer values of $N_m \in [1, N]$. We observe that for integer fractions of the original length (e.g., $N_m = 20, 40, 80$) the fundamental frequency f_0 stays true to the reference case $N_m = 160$ and the straight-line waveguide pitch. In contrast, for other values the f_0 estimation deviates from the reference on average by ~ 4.0 Hz, with a deviation peak encountered at $N_m = 64$ of ~ 25.0 Hz.

The same overall behavior can be seen on Figure 8 (b) and (c) for $N = 179$ and $N = 200$. For the latter we found the same average deviation of ~ 3.6 Hz, and a peak encountered at $N_m = 80$ of ~ 20.0 Hz. In the case of $N = 179$, a prime number, no other value matches the same f_0, but the deviation between the whole range of N_m stays more constrained, with an average of ~ 4.3 Hz and no exceptional peaks.

3.3 VST plugin

We prototyped a VST plugin in JUCE to experiment with possible real-time applications of the algorithm. Figure 9 shows the defined parameters on Ableton Live's default GUI for VST plugins.

As controllable parameters we defined the overall length of the structure N, the length of the middle section N_m as fraction of the overall length, and the reflection coefficient R_0. In addition, we added a simple 1-pole lowpass filter on the closing loop of the original structure as is usually done in physically-informed waveguide modes, and gave the user control to the Damping amount. Finally, an impulse trigger is added to test the impulse response to the structure, but the VST also receives any incoming audio into the merge-cross structure, opening the door for it working as a resonator for noise or any other sound source.

The real-time algorithm shows the same behavior as the experiments previously describe, as expected. The implementation is also relatively cheap in terms of processing and memory usage, which opens the door for its implementation for hardware audio generators and physical user interfaces.

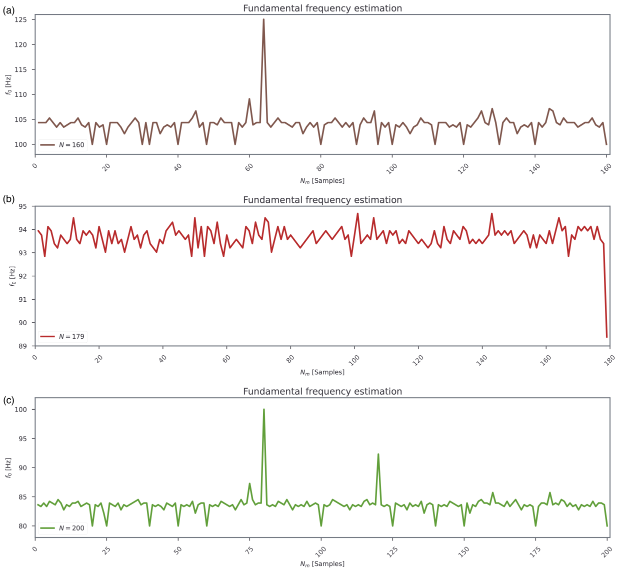

Figure 8: Fundamental frequency estimation for merged-cross waveguide structures with different overall size N and integer values of $N_m \in [1, N]$. (a) $N = 170$. (b) $N = 179$. (c) $N = 200$

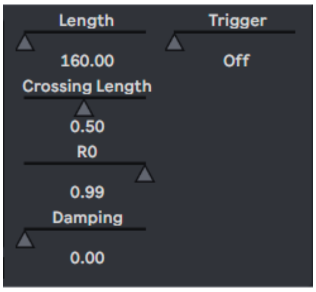
Length 160.00
Trigger Off
Crossing Length 0.50
R0 0.99
Damping 0.00

Figure 9: Ableton Live default GUI for VST plug-ins, showing the real-time controllable parameters of the prototype.

4 Future Work

From a signal processing point of view, the paths to follow are many. Modifications to the merged-cross structure can be added to tune the fundamental frequency and duration of the sound as needed, like adding all-pass filters and fractional delays like in the modifications introduced to the Karplus-Strong algorithm [30]. This would allow us to have independent control of the pitch and timbre of the resulting signal, allowing a more customizable experience for the performer. In addition, further experiments will be done connecting more than one crossing sections in series, and real-time controls for the amount of crossings in the structure. This can give an option of utilizing the knot theory rules to distinct such topologies.

Second, a meaningful connection with a GUI should be implemented to enrich the user experience. The development of the VST plugin with an interface resembling the one presented on Section 2.2 would help us transmit the physical and mathematical inspiration of the algorithm and let the user interact directly with changes in the structure, rather than more abstract parameters.

Finally, the possibility of embedded hardware implementations allows the integration with physical objects and interfaces. The algorithm could be triggered by an actual string, and performers could interact by modifying the actual string length and number of crossings generating the equivalent modifications on the algorithm itself.

5 Conclusion

In this work we have tested a possibility of a new type of sound generator, which was heavily inspired by physical and mathematical properties of knots. We have showed how knots can be interpreted in DSP with sonification of periodic series. And we moved further to physically inspired implementations with waveguide topologies.

We showed that by interpreting crossings as 3-port junctions in our merged-crossing geometry we obtain a richer timbre in comparison to the 2-port version usually used for strings and tubes. In addition, we gain the ability to manipulate the formant of the spectrum by changing a single parameter of the structure and introduce amplitude modulation, all while keeping the pitch relatively stable. Our model expands on the idea of physically-inspired but not physically-restricted digital waveguides, allowing real-time manipulation of its outcome and achieving sound palettes not present on traditional tube or string models.

Finally, we showcased applications of the system as a sound synthesizer, and built functional real-time prototypes for software synthesizers, as well as the base code for the audio processing on audio embedded hardware.

[please click here to see the video attachment for demonstrations]

6 Ethical Standards

This research has been done without any external funding or any other financial support. No other animals, rather than the authors themselves, have been involved in tests and development.

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