Coupled Oscillator Networks: Perspectives on Synchronization and Nonlinear Musical Frameworks

Gyuchul Moon
Institute of Sonology
Royal Conservatoire The Hague
Spuiplein 150, 2511 DG Den Haag
The Netherlands
gyuchulm@gmail.com

ABSTRACT

The central emphasis of the paper is exploring the potential of utilizing the state of a specific system or natural phenomenon as a means of musical expression. This paper delves into applying coupled oscillator networks from sound synthesis to composition. It employs mathematical viewpoints grounded in coupled oscillators, comparing the Kuramoto model and the recently discovered Janus model for simulating musical frameworks by formulating and implementing the simulation of Janus oscillator networks to exploit the perspectives of generative and nonlinear in the composition process. Coupled oscillators implemented with multiple agents can be used at an analytic level with different types of parametric space, and complex behavior depends on each coupling value of the group and each agent's relationship. It can be exploited at various levels of settings in the composition and sound synthesis stages. I outline the way of sound synthesis through the simulation of Janus network oscillators to find the meaning of compositional techniques underlying the self-organized principles.

Author Keywords

Composition, Nonlinear, Sound Synthesis, Coupled Oscillator, Supercollider

CCS Concepts

•**Applied computing** → **Sound and music computing;** Performing arts;

1. INTRODUCTION

In nature, organisms, and the broader ecological system, we encounter a myriad of dynamic and complex processes. Examples include the synchronized movements of bird flocks, the organized activities within ant colonies, and the rhythmic patterns inherent in natural cycles. Intricate, interconnected, and dynamic complex systems characterize the world. These systems have long captivated the curiosity of scientists, philosophers, and artists. Various techniques have been developed in computer music, including nonlinear sound synthesis, system theory, cybernetics, swarm models, physical modeling synthesis, and statistical synthesis processes. This field not only pushes the limits of human capability but also aims to incorporate complexity into the compositional process. The conventional linear mathematical approaches of traditional physics often fail to capture the nonlinear, emergent behavior exhibited by interconnected elements in nature. As the paper [1] pointed out, "In a linear system, the output (effects) is proportional to the input (causes). However, many natural phenomena and systems are intrinsically nonlinear, having no proportional relation between causes and effects".

This paper tries to broaden our understanding of the phenomena that exist in nature, especially synchronization and nonlinear behavior. The Kuramoto model is widely adopted to explain and analyze the synchronicity of nature. However, perspectives of the sound synthesis and compositional process should be discussed to generate the appropriate amount of dynamics within a system. The paper examines two coupled oscillator networks, the Kuramoto coupled oscillator networks and the recently discovered Janus coupled oscillator networks. As follows, I will discuss the processes of composition and sound synthesis, focusing particularly on how a reductionistic perspective, grounded in formulas, expands the holistic points of view in composition to generate comprehensive dynamics by seamlessly incorporating emergence and contingency. Furthermore, at the end of the paper's conclusion, it presents some practical examples derived from the multi-agent simulation and its application discussed in the paper.

2. KURAMOTO COUPLED OSCILLATOR NETWORKS

2.1 Mathmatical Description

The Kuramoto model was originally motivated by the phenomenon of collective synchronization, in which an enormous system of oscillators spontaneously locks to a common frequency despite the inevitable differences in the natural frequencies of the individual oscillators. The Kuramoto model describes a large population of coupled limit-cycle oscillators whose natural frequencies are drawn from some prescribed distribution [2]. It was introduced by the Japanese physicist Yoshiki Kuramoto in 1975 [3]. The Kuramoto model is widely used in various fields, including physics, biology, neuroscience, and engineering, to understand the collective behavior of coupled oscillatory systems. The Kuramoto model is commonly depicted using a circle where

NIME'24, 4–6 September, Utrecht, The Netherlands.

each point on the circumference corresponds to an oscillator, with N oscillators in total. The angular position of each point, denoted as θ_i, signifies the phase of the perspective oscillator, and the circle illustrates the full range of possible position values, typically spanning from 0 to 2π radians. The governing equation for each oscillator's phase is shown for the ensemble in Equation. (1).

$$\dot{\theta}_i = \omega_i + \frac{K}{N}\sum_{j=1}^{N} sin(\theta_j - \theta_i - \alpha_0) \tag{1}$$

θ_i is the phase of i_{th} Oscillator and $\dot{\theta}_i$ is the derivative of phase to time. ω_i is the natural frequency of the oscillator i. in a population of N oscillators. K is the coupling factor, and the term of $sin(\theta_j - \theta_i)$ is the phase response function that determines the interaction between each oscillator in a group. An effective modification involves introducing a phase offset or "frustration" parameter, α_0, within the phase response function.

It is convenient to imagine a swarm of points running around a unit circle in the complex plane in Fig. (1). The mean-field approximation assumes that as $N \to \infty$, the oscillators become effectively decoupled, and the overall behavior is captured by the mean-field order parameter $Re^{i\psi}$. The order parameter R represents the phase coherence, and ψ represents the average phase of the ensemble. The order parameter R and average phase ψ are defined as follow Equation. (3).

$$Re^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_i} \tag{3}$$

Equation. (1). can be rewritten neatly regarding the order parameter as follows. Multiply both sides of the order parameter equation by $e^{-i\theta_i}$, and equating imaginary parts yields then equation(1) can be written as Equation (4)

$$\dot{\theta}_i = \omega_i + KR\sin(\psi - \theta_i) \tag{4}$$

The mean-filed character of the model becomes obvious. Each oscillator appears to be uncoupled from the others [2]. However, oscillators interact only through the mean-field quantities R and ψ. In particular, the phase $\dot{\theta}_i$ is pulled toward the mean phase ψ rather than the phase of any individual oscillator. Moreover, the effective strength of the coupling is proportional to the coherence R. This proportionality sets up a positive feedback loop between coupling and coherence. As the population becomes more coherent, R grows, so the effective coupling K increases, which tends to become synchronized over time.

2.2 Sound Synthesis with Kuramoto Model

This paper [4] outlined additive sound synthesis approaches. Implementing the sound synthesis model with additive synthesis is straightforward with a Kuramoto oscillator using a multi-agent method. Position values of each agent are used to map the frequency as the position of an angular value representing the oscillator's frequencies. Its results synchronization of sinusoidal waves depends on each agent's position value as of its frequencies.

In the Kuramoto model, each agent in the group sits within a specific circle radius. Each agent's position is directly mapped on the particular frequencies. One of the primary focuses of the paper is a specific system with a holistic point of view of the system to generate more synchronization and nonlinear sound synthesis methods. Sound

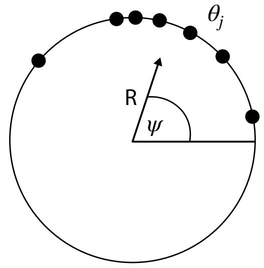

Figure 1: "Ensemble of coupled oscillators represented in a circle map as a "swarm of points" moving about a circle [2].

synthesis with the Kuramoto model is a prominent solution for coupling and synchronizing the sounds of oscillators. However, from a non-linear behavioral perspective, the Kuramoto model has apparent limitations for musical perspectives that create emergence and contingency within a system. The status of the coupled and decoupled agent can only be determined by the coupling strength K, which means it has limited status expression within a model. Therefore, This paper [4] tries to solve this issue by adding more dynamics within a system, adding specific distributions with the Equation. (4), reformulating it as an Equation. (5).

$$\dot{\theta}_i = \omega_i + \Lambda_e(\theta_i) + KR\sin(\psi - \theta_i) \tag{5}$$

The system equations are in a trade-off between a state of synchronicity or an incoherent state. $\Lambda_e(\theta_i)$ is a distribution to add more emergence or external forces to generate more characteristics, like Sawtooth interaction function [5], or simple Gaussian distributions. In this case, the system aligns with specific frequencies but can add more forces with α_0 between $0 < \alpha < 2\pi$ as in an incoherent state.

However, further discussion is needed regarding the perspectives of dynamics for composition. The system exhibits a phase transition if the coupling strength exceeds a certain threshold. Some oscillators spontaneously synchronize, while others remain incoherent [2]. Even in this bifurcation between synchronization and incoherence of the group, it is constrained to accessing points of the sound synthesis process with the nonlinear characteristic. Adding more distribution or force within a system is still defined as two states between synchrony and incoherence, highlighting the distinct limitation of creating emergent sound characteristics.

3. JANUS OSCILLATOR NETWORKS

3.1 Multifaceted Janus Oscillator Networks

Initially introduced in 1975 by Kuramoto [3], the Kuramoto model has significantly advanced our comprehension of coupling and synchronization phenomena. Despite its considerable contributions, delving into sound synthesis's nonlinear and complex behavioral aspects requires further discussion. The tradition of physics has been to describe complex behavior using simple mathematical models. However, most of the natures modeled by the traditional mathematical physics methods are linear [6]. Even in a nonlinear topic with a synthesis and compositional model with ODE, like Chaos, the complex behavioral complexity is emergent rather than explicitly coded in the model [7]. From this perspective, the Kuramoto model contains the novelty of equations and the model's behavior. However, more dimensions

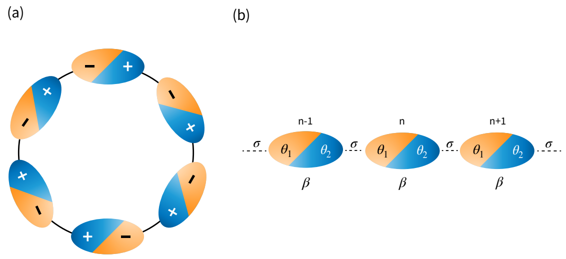

Figure 2: **(a) Model networks of phase-phase oscillators, in which phase oscillators with alternating frequencies are sine-coupled to their nearest neighbors on the ring. Each pair of plus-minus oscillators as a Janus Oscillator results in a symmetric system of oscillators [7]. (b) Describing the mathematical model to depict the relationship with neighbors, each node is equipped with two internally coupled oscillators, where β is internal coupling strength, σ is external coupling strength**

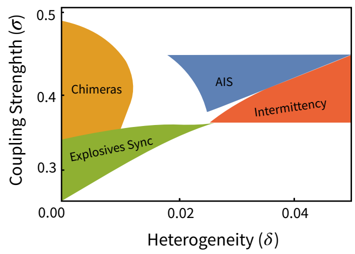

Figure 3: The various phenomena exhibited by a ring of N = 50 Janus oscillators as the coupling strength and heterogeneity across the Janus oscillators are varied [7].

need to be discussed regarding the dynamic and emergent characteristics of sound synthesis levels.

Recent research has unveiled novel phenomena in network synchronization. (1) Chimera states [8] with coexisting incoherence and synchrony in identically coupled identical oscillators, (2) explosive synchronization [9] in which the transition to synchronization becomes subcritical (hence abrupt) and hysteretic, and (3) asymmetry-induced synchronization [10, 11], which is a partial converse to the symmetry breaking exhibited by chimera states, in which either the oscillator or their coupling need to be nonidentical for synchronization. The complex behaviors of each model are unequivocally rather emergent than implicitly coded in the model, and each model requires the design of a specific system. However, The paper [7] elaborates on the co-occurrence chimera state, explosive synchronization, and a new form of AIS in a class with a simple oscillator network. The dynamical units in these networks are two-dimensional phase-phase oscillators, illustrated in Fig. (2) (a). The name is inspired by the homonym two-faced god of Roman mythology and reflects the two-dimensional character of the isolated oscillator. Each unit phase consists of a Kurmoto oscillator whose natural frequency has the same absolute value but opposite sign to the frequency of its counterparty.

3.2 Mathematical Description

In the Janus model, each oscillator can be represented by a clock with two arms: a minute arm and an hour arm. The positions of these arms are interdependent. Consider a situation comprising numerous dual-arm clocks, each with its own natural frequency for the minute and hour arms. In the Janus model, the oscillators interact with each other based on the relative positions of their minute and hour arms, creating a more complex and interconnected system than the Kuramoto model. The Kuramoto model can be considered a reduction version of the Janus model. The Kuramoto model focuses only on the hour arm (or the phase) of each oscillator, and the coupling between oscillators depends solely on the difference between their hour arm positions. The Janus model, on the other hand, takes into account the relationship between the minute and hour arms within each oscillator, as well as the coupling between the oscillators themselves. It adds complexity to the Janus model, allowing for more intricate and diverse collective behaviors to emerge. The synchronization patterns in the Janus model can be influenced not only by the coupling strength between oscillators but also by the internal dynamics within each oscillator, determined by the relationship between its minute and hour arms.

$$\dot{\theta}_i^1 = \omega_i^1 + \beta \ sin(\theta_i^2 - \theta_i^1) + \sigma \ sin(\theta_{i-1}^2 - \theta_i^1) \qquad (6)$$

$$\dot{\theta}_i^2 = \omega_i^2 + \beta \ sin(\theta_i^1 - \theta_i^2) + \sigma \ sin(\theta_{i+1}^1 - \theta_i^2) \qquad (7)$$

For $i = 0, ..., n-1$, where subscripts of Janus oscillator superscripts indicate each oscillator's index. The parameters ω_i^1 and ω_i^2 are natural frequencies of oscillator i. β is the internal coupling strength between the phase-oscillator components of each Janus Oscillator, and σ is the external coupling strength between oscillators in different nodes [7]. Unlike the Kuramoto model, which is divided into two states depending on the coupling values of the group K, Jauns oscillator networks suggest a further understanding of the coupling and incoherent status. Janus oscillator is a two-dimensional phase-phase oscillator in which each component has a distinct natural frequency. As illustrated Fig 2. (b).

When ω, natural frequency, is a constant of each node within a system, then Equation 6 and 7 is three-dimensional with coordinates axis ω, σ and β. To simplify, in this paper, average frequency is regarded as $\bar{\omega} = 0$. However, the space can be substantially reduced without the loss of generality. When either the σ or β are excessively large (exceeding the critical value of $\omega/2$, at which an isolated pair of oscillators would synchronize), the two-phase components of each oscillator phase-lock. Consequently, the dynamics can be simplified to a ring of single-phase oscillator nodes. We set the intrinsic frequency to explore the dynamics of interest in the ring. $\omega = 1$, and coupling value $\beta = \omega/4$ while varying the external coupling strength in the range of $0 \leq \sigma \leq 0.6$ in Fig. (3). Despite the simplicity of the topology and the Janus oscillator model itself, it exhibits a dynamic behavior encompassing the co-occurrence of several dynamical patterns.

4. JANUS MODEL AS NONLINEAR MUSICAL FRAMEWORKS

4.1 Methodology

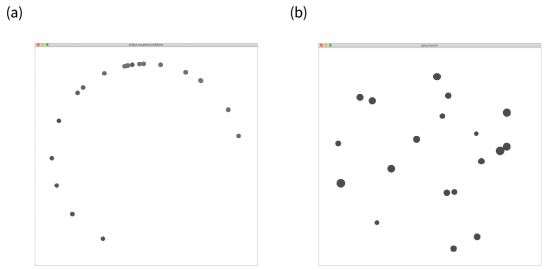

Figure 4: (a) Simulation of Kuramto model, $N = 20$, $K < K_c$ (b) Simulation of Janus model $N = 20$, $\beta = 0.1$, $\sigma = 0.3$. (a) and (b) both run in a same radius of the circle.

While many sonic devices typically adhere to conventional sound synthesis approaches, the Janus model, in the perspective of behavior, exemplifies emergent and contingent behavior with a real-time simulation model. A system is a tangible or conceptual entity in the real world, while a model is a simplified or abstract representation of that system. Badiou exemplifies and describes the conditions of the models; "The model will always be true. The simplest possible model is derived exclusively from the facts under consideration. And that the artificial object explains all the empirical facts under consideration" [12]. In an implementation model with formalized simulations, the paper aims to implement a contingent, non-linear sound and composition process as a model that bridges the understandable and the perceptible.

The formula of the Janus model would not be merely regarded as a simple synthesis method. The rationale extends beyond the diverse state expressions within the formula through simulation. The exploration aims to ascertain whether such non-linear behavior can be observed from a holistic point of view with a musical perspective. In a musical framework, the Janus model has been implemented through multi-agent simulation, serving as a dynamic composition tool to apply parametric changes in the ω, β, σ space over time. Even in a specific state where parameters are fixed, It also delves into the possibilities of unique musical expression within specific states characterized by fixed parameters as illustrated in Fig. (3).

4.2 Additive Synthesis

Simulation with a coupled oscillator follows a trajectory corresponding to a defined radius circle. The fundamental synthesis method involves treating the coupled oscillator group as a collection of oscillators, where each oscillator's position is directly mapped. The Euler method is employed to generate sounds with dependent frequencies based on the positions of the oscillators. For the Kuramoto model, each oscillator is in a group with a specified radius and a circle, but for the Janus model, it also binds the specific radius of the circle. However, It depends on the coupling values of ω, β and σ to define the whole behavior of the group and the set of parameters generates parametric space. See Fig. (4). Kuramoto's model can be considered a special case of the Janus model. External coupling values σ are significantly huge, and the behavior is similar to the behavior of the Kuramoto model. In the Janus model, the behavior of each oscillator is inherently defined by certain coupling parameters. Sound generation occurs based on fixed frequencies corresponding to specific states. However, the primary emphasis here lies in the dynamic setting of the parametric space through simulation, allowing real-time adjustments beyond a fixed state. This real-time adaptability allows dynamically altering parameters, enhancing the potential for capturing diverse and nonlinear sonic characteristics based on their values.

The Kuramoto model only defined the group's status with coupling values, K, but the Janus model defined the four status types with the agents' value and internal and external coupling values. So, Heterogeneity δ and strength of the Coupling strength σ define the four types of status of the system. 1) Chimera state, 2) Explosive Sync state, 3) AIS state, 4) Intermittency state. One of interest for the sound synthesis process is that even if we choose the sinusoidal waves, it is not just a reprint of the coupling phenomena but also, from the perspectives of status, exhibits much more diversity in a system. The Janus model explains synchronization phenomena that are expressed differently depending on the corresponding parameter values and has the advantage of the emergence of nonlinearity within a system, which is implemented through simulation.

4.3 Two or Multi-Dimensional Sonic Morphological Methods

In the Janus oscillator model, the parametric space, defined with ω, β and σ, reveals synchronization in a group and shapes more dynamics and nonlinearity in a sound synthesis process. The group tendency is defined by frequency oscillators with a distinct behavioral pattern and a corresponding positional value. From the simulation perspective, which possibly changes the parameter in real-time, This flexibility allows for exploring a diverse sonic landscape, offering a spectrum of timbre qualities that the parameters can manipulate. It means shaping the timbre characteristics based on the unique states of each oscillator illustrated in Fig. (3).

The Janus model, through its emphasis on simulations rooted in characteristic states rather than exclusively relying on empirical outcomes, presents a perspective that amplifies the accessibility of self-organizing and emergent behavior within the musical context. Conventional terminologies associated with self-organization frequently link the concept to the system-based or cybernetics concept of music. However, the Janus model could broaden the meaning of self-organization, simulating characteristic states representative of distinctive behavioral patterns and exploring self-organizing and emergent phenomena.

From the perspectives of nonlinearity and generative musical frameworks, it can be considered a two- or multi-dimensional space for morphological effects or the generation of much more musical results. Each oscillator-governed equation of the Janus model contains more possibilities than generating sound morphology with random mapping or matrix representation, which creates intriguing musical behavior or emergence underlying the self-organized principle. The simulation governs all oscillators and makes the parametric space, for instance, x-y morph, with different effects, like x maps the amount of filter, y maps the amount of frequency shift, or x maps the amplitude value, y maps the pitch shift. A simple setting for the map of the musical parameter can generate enormous sonic possibilities for usage.

Also, most of my interests in the paper, both sound synthesis and compositional process, are related to the Janus model. But when we consider using diffrent types of sound synthesis sources like Pulsar synthesis [13], which can be adjusted with the Pulsaret, which are parameters of Pulsar synthesis. The position of the Janus oscillator is mapped to dynamically adjust certain sound characteristics within Pulsar synthesis. The integration of Pulsar synthesis introduces a dynamic interplay between sound synthesis and a rhythmic generator, offering a versatile foundation for various synthesis methods. In Pulsar synthesis, when the fundamental frequency descends below 20 Hz, sonic characteristic gives rise to a series of individually audible pulses, effectively blurring the traditional distinctions between rhythm and pitch. Through the simulation of changing Pulsaret over time with the Janus Model, The unique characters expand the sonic possibilities, creating intricate and evolving textures that challenge conventional notions of rhythmic and melodic elements in music.

4.4 Spatial Compositional Tools

In the paper [14], Chowning work on the simulation of moving sound sources discusses the essential localization cues that help listeners perceive the spatial position of a sound source in an enclosed space. According to Chowning, the listener requires two main types of information to accurately locate a sound source: angular location and distance cues. The cues for the angular location are (1) the different arrival time or delay of the signal at the two ears when the source is not centered before or behind the listener and (2) the pressure-level differences of high-frequency energy at the two ears resulting from the shadow effect of the head when the source is not centered. The cues to the distance of a source from a listener are (1) the ratio of the direct energy to the indirect or reverberant energy where the intensity of the direct sound reaching the listener falls off more sharply with distance than does the reverberant sound and (2) the loss of low-intensity frequency components of a sound with increasing distance from the listener.

In spatial composition, the placement and movement of sounds in space are crucial elements in shaping the listener's experience. Traditional approaches to spatial composition often rely solely on the manual placement of sounds. However, The Janus model provides a way to create spatial compositions that emerge from the interactions and self-organizing behavior of the sound sources themselves. It can set up a system where the spatial arrangement of sounds emerges naturally from the collective behavior of the oscillators underlying the self-organized principles. Current spatialization methods are close to the generative model for shaping spatial images, which means they are not directly mapped with manual localization. The behavioral patterns are only determined by the parameters of the simulations. The objective of the research was to explore some of the collective behaviors that could be implemented in the compositional process and also how this approach can be used in the electroacoustic compositional methods.

The qualities of nonlinearity of the model are that each node in the Janus model does not just rotate in a bonded circle. This means that spatial composition has more possibilities for interesting spatial patterns within a space to generate angular and distance cues, as Chowning pointed out. The Janus model can generate interesting local and global oscillations in a spatial setup depending on the parameter, as illustrated in Fig. (5). Combining the simulation with musically mapped parametric space would generate nonlinear sonic sculpturally based on the different types of state expressions as made spatial images. Certain spatial compositional qualities are contingent upon the physical model. Sound, as a manifestation of energy, propagates through space and is intricately tied to the movement of matter. Physical forces govern this movement, and the intricate interplay of these forces can be effectively simulated with the Janus model in a compositional process.

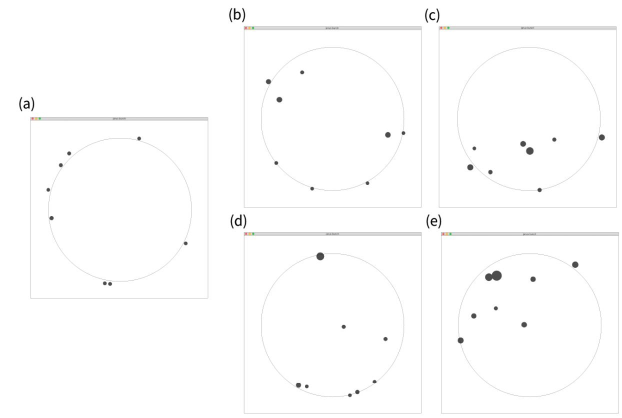

Figure 5: (a) For $N = 8$ in the Janus Model, phase-locking situation. Each node adheres closely to the unit circle within the simulation. (b) to (e) Likewise, for $N = 8$, diverse behavior patterns emerge and evolve over time with the same parameter settings.

5. CONCLUSION

The paper discussed sound synthesis and musical frameworks with coupled oscillator networks. One of my interests in the paper is that synchronization and nonlinear exhibits in nature can be used as musical frameworks through multi-agent real-time simulation.

I believe complexity can not merely be the aesthetic of the composition. In this manner, I was trying to avoid the meaning of nonlinearity to describe complexity only at empirical levels. The paper aims to establish the groundwork for sound generation and synthesis methods in music, characterized as either emergence or nonlinear. So, through the equations representing an abstraction of a real-world entity, accessing a holistic point of view through the lens of emergence and nonlinear musical behavior is an essential perspective of the process. The processes focus not only on mirroring these phenomenological behaviors in music but also on composing audible matter that exists between the intelligible and the sensuous.

The paper examined the meaning of state expression as a compositional process. Simulation with Janus oscillator networks not only explains synchrony and nonlinear behavior related to each element in a group but also defines the different levels of parametric spaces defined by the status expressions. In realizing these intentions, the paper draws upon the possibilities through recently discovered Janus coupled oscillator networks, examining sound synthesis, compositional possibilities with morphological level, and the potential of the spatial compositional process. The paper sidestepped the trajectory of technological determinism, instead delving into how interaction with parametric space in Janus coupled oscillator networks can broaden the meaning of the compositional process through real-time simulation.

6. LINKS

Sound examples referenced in this paper are accessible through the following link:

https://bit.ly/3StC3RZ

7. ETHICAL STANDARDS

As a composer and artist, I am committed to upholding ethical principles in my artwork. The research process carries unique ethical challenges and responsibilities. The research procedures are not intended for financial profits. Instead, research explores the possibilities of real-time sound work, motivated by my curiosity and desire to experiment with artistic directions. In the research process, I strive to maintain the following ethical principles: 1. Authenticity and Integrity 2. Transparency and accountability, 3. Social and Environmental Responsibility.

8. REFERENCES

[1] D. Sanfilippo and A. Valle, "Feedback Systems: An Analytical Framework," *Computer Music Journal*, vol. 37, pp. 12–27, June 2013.

[2] S. H. Strogatz, "From Kuramoto to Crawford: exploring the onset of synchronization in populations of coupled oscillators," *Physica D: Nonlinear Phenomena*, vol. 143, pp. 1–20, Sep 2000.

[3] Y. Kuramoto, "Self-entrainment of a population of coupled non-linear oscillators," in *International Symposium on Mathematical Problems in Theoretical Physics* (H. Araki, ed.), (Berlin, Heidelberg), pp. 420–422, Springer Berlin Heidelberg, 1975.

[4] N. Lem, "Sound in multiples: Synchrony and interaction design using coupled-oscillator networks," *Conferance: Sound and Music Computing Conferance*, Nov 2019.

[5] J. Snyder, A. Zlotnik, and A. Hagberg, "Stability of entrainment of a continuum of coupled oscillators," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 27, pp. 103–108, Oct. 2017.

[6] F. Heylighen, *The Science of Self-organization and Adaptivity.* EOLSS Publishers Co Ltd, 2002.

[7] Z. G. Nicolaou, D. Eroglu, and A. E. Motter, "Multifaceted dynamics of Janus oscillator networks," *Physical Review X*, vol. 9, pp. 011–017, Jan 2019.

[8] D. M. Abrams and S. H. Strogatz, "Chimera states for coupled oscillators," *Physical Review Letters*, vol. 93, Oct. 2004.

[9] J. Gómez-Gardeñes, S. Gómez, A. Arenas, and Y. Moreno, "Explosive synchronization transitions in scale-free networks," *Phys. Rev. Lett.*, vol. 106, pp. 128–701, Mar 2011.

[10] T. Nishikawa and A. E. Motter, "Symmetric states requiring system asymmetry," *Phys. Rev. Lett.*, vol. 117, pp. 114–101, Sep 2016.

[11] Y. Zhang, T. Nishikawa, and A. E. Motter, "Asymmetry-induced synchronization in oscillator networks," *Phys. Rev. E*, vol. 95, pp. 062–215, Jun 2017.

[12] A. Badiou, Z. Fraser, and T. Tho, *The Concept of Model: An Introduction to the Materialist Epistemology of Mathematics.* Re.Press, 2007.

[13] C. Roads, *Microsound.* The MIT Press, 2002.

[14] J. M. Chowning, "The simulation of moving sound sources," *Computer Music Journal*, pp. 48–52, 1977.